ACL2(ml): Machine-Learning for ACL2

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Outline

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Some Challenges in ACL2

- Size of ACL2 library stands on the way of efficient knowledge reuse.
- Manual handling of proofs, strategies, libraries becomes difficult.
- Coordination of team-development can be hard.
- Comparison of proof similarities.
- Discovery of auxiliary lemmas can be difficult.

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- Symbolic methods (Proof planning, lemma discovery) can conceptualise but have limitations.

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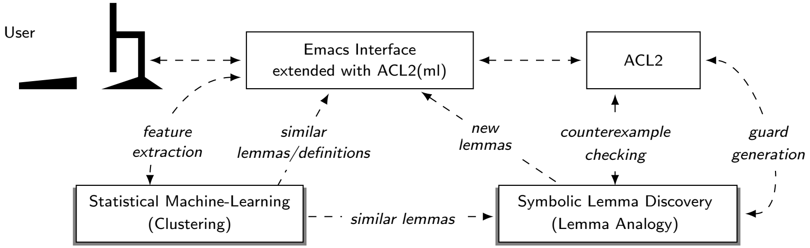
- Statistical methods can discover patterns in proofs but are weak for conceptualisation.
- Symbolic methods (Proof planning, lemma discovery) can conceptualise but have limitations.
- Combination of statistical and symbolic methods:
  - Statistical methods can take advantage of symbolic methods to conceptualise results.
  - Symbolic tools can use statistical results for efficient lemma discovery.

Outline

ACL2(ml)



F.1. works on the background of Emacs extracting some low-level features from ACL2 definitions and theorems.

F.2. automatically sends the gathered statistics to a chosen machine-learning interface and triggers execution of a clustering algorithm of user's choice;

F.3. does some post-processing of the results and

- **F.3.a** displays families of related proofs (or definitions) to the user.
- **F.3.b** uses the families of related proofs to discover new lemmas.

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Extracting features from ACL2

- Feature extraction:

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  - We extract features directly from term trees of ACL2 terms.

Definition (Term tree)

A variable or a constant is represented by a tree consisting of one single node, labelled by the variable or the constant itself. A function application $f(t_1, \dots, t_n)$ is represented by the tree with the root node labelled by f, and its immediate subtrees given by trees representing $t_1, \dots, t_n$.

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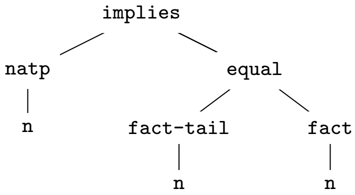
```
(implies (natp n) (equal (fact-tail n) (fact n)))
```



ACL2(ml) term tree matrices

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Definition (Term tree depth level)

Given a term tree T, the *depth* of the node t in T, denoted by *depth(t)*, is defined as follows:
$-$ $depth(t) = 0$, if t is a root node;
$-$ $depth(t) = n + 1$, where n is the depth of the parent node of t.

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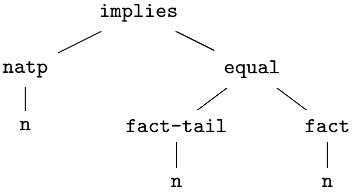
Definition (ACL2(ml) term tree matrices)

Given a term tree T for a term with signature Σ, and a function $[.] : \Sigma \to \mathbb{Q}$, the ACL2(ml) term tree matrix M_T is a 7×7 matrix that satisfies the following conditions:
– the $(0, j)$-th entry of M_T is a number $[t]$, such that t is a node in T, t is a variable and $depth(t) = j$.
– the (i, j)-th entry of M_T $(i \neq 0)$ is a number $[t]$, such that t is a node in T, t has arity $i + 1$ and $depth(t) = j$.

An example



	variables	arity 0	arity 1	arity 2
$td0$	0	0	0	[implies]
$td1$	0	0	[natp]	[equal]
$td2$	[n]	0	[fact-tail]::[fact]	0
$td3$	[n]::[n]	0	0	0

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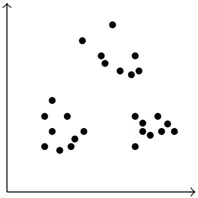

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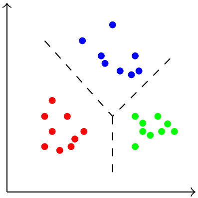

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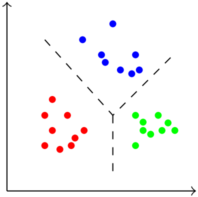

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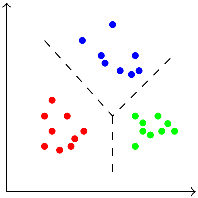

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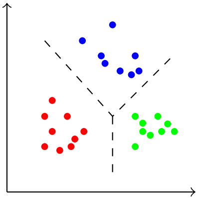

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Recurrent clustering

Three kinds of function symbols:

- Built-in functions: predefined value.
- Variables: based on the De Bruijn index.
- Functions defined on terms of other functions: recurrent clustering process.
    - Recursive and mutually-recursive function occurrences have a fixed value.

Demo

- Finding similar theorems across libraries.
- Obtaining more precise clusters.
- Finding similar definitions across libraries.

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Terminology:

- Target Theorem (TT): the theorem that we want to prove.
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- Source Lemma (SL): a user-supplied lemma to prove ST.

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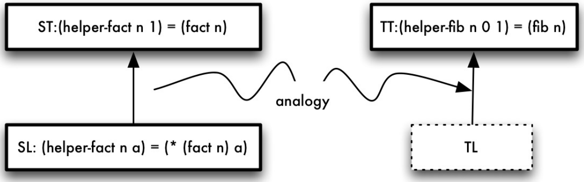

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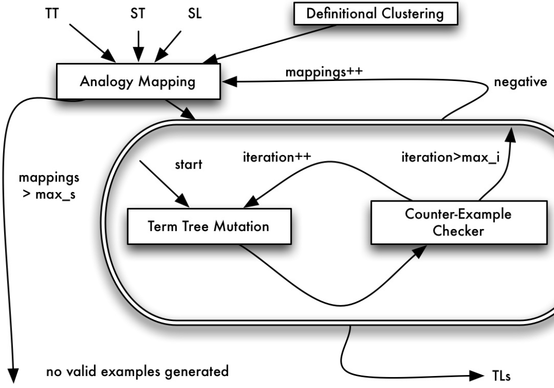
Overview of the process



Using guards to generate preconditions

- Using the lemma analogy tool, ACL2(ml) generates the following suggestion:

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(equal (helper_fib n j k)
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- Guards are optional and several functions do not include them.
- ACL2 recommendation for novices: "novices are often best served by avoiding guards".
- Solution: compute recursively the guards of a function f.

Using guards to generate preconditions

```
(defun helper_fib (n j k)
     (if (zp n) j (if (equal n 1) k (helper_fib (- n 1) k (+ j k)))))

  * zp ->  (natp x)
  * equal -> t
  * + -> (and (acl2-numberp x) (acl2-numberp y))
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Guards generated for `helper_fib` $\rightarrow$

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(and (natp n) t (and (acl2-numberp n) (acl2-numberp 1))
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```
(defthm helper_fib_theta_fib
   (equal (helper_fib n j k)
          (+ (* (theta_fib (- n 1)) j) (* (theta_fib n) k))))
```

Guards:

```
(and (integerp n) (not (< n 0)) (acl2-numberp j) (acl2-numberp k)
     (not (< (+ -1 n) 0)))
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Demo

- Lemma discovery.
- Guard generation.

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Conclusions

- ACL2(ml) combines statistical machine learning (detection of patterns) with symbolic techniques (generation of lemmas).
- ACL2(ml) is different to other tools:
  - its methods of generating the proof-hints interactively and in real-time;
  - its flexible environment for integration of statistical and symbolic techniques.

Further work

- **Reimplement ACL2(ml) as ACL2 book.** All ACL2(ml) modules are currently implemented in Emacs Lisp.
- **Use of information generated by failed proof-attempts.**
- **Different patterns.** Statistical ACL2(ml) groups in the same clusters theorems whose lemmas cannot be mutated to generate any useful lemma.
- **Smaller lemmas.** The lemma analogy tool currently only adds term structure; therefore, it cannot generate smaller lemmas.
- **Conditional lemmas.** Discovering appropriate conditions for generated lemmas is a difficult problem for theory exploration systems.
- **New definitions.** Another big challenge in lemma discovery is the invention of new concepts.

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How is the function [.] defined?

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Definition (Function [.])

Given the nth term definition of the library (call the term t), a function [.] is inductively defined for every symbol s in t as follows:
$-$ $[s] = i$, if s is the ith distinct variable in t (formulas are implicitly universally quantified);
$-$ $[s] = -[m]$, if t is a recursive definition defining the function s with measure function m;
$-$ $[s] = k$, if s is a function imported from CLISP; and $[s] = k$ in the figure below;
$-$ $[s] = 5 + 2 \times j + p$, where C_j is a cluster obtained as a result of definition clustering with granularity 3 for library definitions 1 to $n-1$, $s \in C_j$ and p is the proximity value of s in C_j.

$*$ Type recognisers (r = {symbolp, characterp, stringp, consp, acl2-numberp, integerp, rationalp, complex-rationalp}): $[r_i] = 1 + \sum_{j=1}^{i} \frac{1}{10 \times 2^{j-1}}$ (where r_i is the ith element of r).

$*$ Constructors (c = {cons, complex}): $[c_i] = 2 + \sum_{j=1}^{i} \frac{1}{10 \times 2^{j-1}}$.

$*$ Accessors (a^1 = {car, cdr}, a^2 = {denominator, numerator}, a^3 = {realpart, imagpart}): $[a_i^j] = 3 + \frac{1}{10 \times j} + \frac{i-1}{100}$.

$*$ Operations on numbers (o = {unary-/, unary–, binary-+, binary-*}): $[o_i] = 4 + \sum_{j=1}^{i} \frac{1}{10 \times 2^{j-1}}$.

$*$ Integers and rational numbers: $[0] = 4.3$, $[n] = 4.3 + \frac{|n|}{10}$ (with $n \neq 0$ and $|n| < 1$) and $[n] = 4.3 + \frac{1}{100 * |n|}$ (with $n \neq 0$ and $|n| > 1$).

Analogy mapping

Definition (Analogy Mapping $\mathcal{A}$)

For all symbols $s_1, \ldots, s_n$ occurring in the current ST, the set of admissible analogy mappings is the set of all mappings $\mathcal{A}$ such that
- $\mathcal{A}(s_i) = s_i$ for all shared background symbols; otherwise:
- $\mathcal{A}(s_i) = s_j$ for all combinations of $i, j \in 1 \ldots n$, such that s_i and s_j belong to the same cluster in the last iteration of definition clustering.

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Example

For our running example, the shared background theory includes symbols {+, *,-, 1, 0}. We thus get a mapping:
$\mathcal{A} = \{$`fact` $\mapsto$ `fib`, `helper-fact` $\mapsto$ `helper-fib`, `+` $\mapsto$ `+`, `1` $\mapsto$ `1`,...$\}$

Term tree mutation

Term tree mutation consists of three iterations:

- Tree reconstruction.
- Node expansion.
- Term tree expansion.

Tree reconstruction

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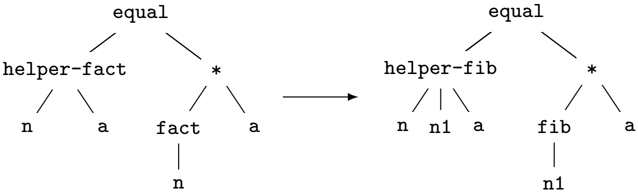

Node expansion

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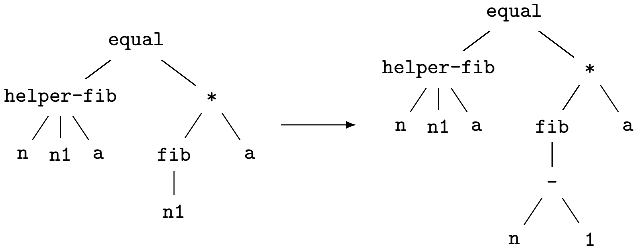

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